UGEB2530 Game and strategic thinking
Quiz 1

Name:______________________________ID:________________Marks:________/40

Time allowed: 60 mins Answer all questions.

1. (4 marks) Circle all pure Nash equilibria of the games.

   (a) $\begin{pmatrix} (4,3) & (-1,0) \\ (2,-1) & (1,2) \end{pmatrix}$

   (b) $\begin{pmatrix} (1,-4) & (-1,-1) & (3,1) \\ (4,4) & (5,2) & (2,-3) \end{pmatrix}$

2. (4 marks) There are 4 cards with numbers 1,1,2,4 printed on them respectively.

   (a) If one card is chosen at random, find the expected value of the number.

   Expected Value $= \frac{1}{4}\times 1 + \frac{1}{4}\times 1 + \frac{1}{4}\times 2 + \frac{1}{4}\times 4$

   $= 2$

   (b) If two cards are chosen at random without replacement, find the expected value of the sum of the two numbers.

   $E = 2\times\frac{1}{2}\times\frac{1}{3} + 3\times\frac{1}{2}\times\frac{1}{3} + 3\times\frac{1}{4} + \frac{2}{3} + 6\times\frac{1}{4}\times\frac{1}{3}\times 2 +$

   $5\times\frac{1}{2}\times\frac{1}{3} + 5\times\frac{1}{4}\times\frac{2}{3}$

   $= 4$

3. (4 marks) Circle all saddle points of the following game matrices.

   (a) $\begin{pmatrix} 3 & 1 & 4 & 3 \\ 5 & 3 & 7 & 4 \\ 1 & 0 & 3 & 2 \end{pmatrix}$

   (b) $\begin{pmatrix} 4 & -1 & -4 & -3 \\ -2 & 3 & -2 & -4 \\ 1 & 0 & 2 & -1 \\ -3 & 2 & -2 & -3 \end{pmatrix}$

4. (10 marks) Michael has a spade(♠) 2 and a heart(♡) 5. Nelson has a diamond(◇) 4 and a club(♣) 4. Each of them chooses a card and they show the cards simultaneously. If the two cards are of the same colour, Michael pays Nelson the sum. If the two cards are of different colours, Nelson pays Michael the sum.

   (a) Write down the payoff matrix for Michael.

Nelson

Micheal	◇4	♣4
♠2	(6,-6)	(-6,6)
♡5	(-9,9)	(9,-9)

payoff matrix is $\begin{pmatrix} 6 & -6 \\ -9 & 9 \end{pmatrix}$

   (b) Suppose Michael chooses spade 2 with a probability of 0.2 and Peter chooses diamond 4 with a probability of 0.4. Find the expected payoff to Michael.

Expected payoff of Micheal

$$= (0.2 \quad 0.8) \begin{pmatrix} 6 & -6 \\ -9 & 9 \end{pmatrix} \begin{pmatrix} 0.4 \\ 0.6 \end{pmatrix}$$

$$= 1.2$$

(c) What is the best strategy of Michael if Nelson chooses club 4 with a probability of 0.6?

assume Micheal use strategy (x, y)

The payoff of him $= (x, y)\begin{pmatrix} 6 & -6 \\ -9 & 9 \end{pmatrix}\begin{pmatrix} 0.4 \\ 0.6 \end{pmatrix}$

$= (x, y)\begin{pmatrix} -1.2 \\ 1.8 \end{pmatrix}$

So the best strategy is $(0, 1)$

(d) Find the optimal strategies for Michael and Nelson.

by oddment method.

$\begin{pmatrix} 6 & -6 \\ -9 & 9 \end{pmatrix}$ $\begin{matrix} 12 \\ -18 \end{matrix}$ $\times$ $\begin{matrix} 18 \\ 12 \end{matrix}$ $\begin{matrix} \frac{3}{5} \\ \frac{2}{5} \end{matrix}$

$\begin{matrix} 15 & -15 \\ 15 & 15 \end{matrix}$ $\times$

$\frac{1}{2}$ $\frac{1}{2}$

So the maximin strategy for M is $\left(\frac{3}{5} \ \frac{2}{5}\right)$

the minimax strategy for Nelson is $\left(\frac{1}{2} \ \frac{1}{2}\right)$

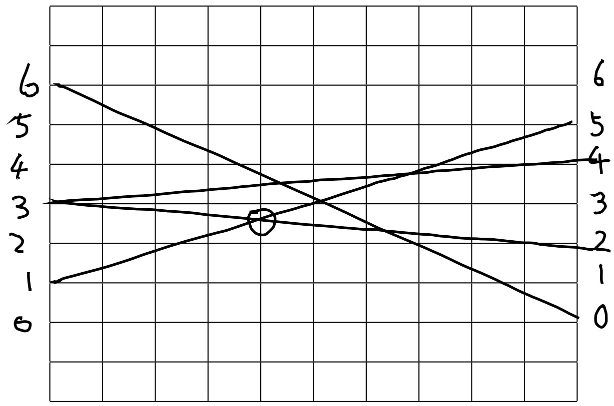
(e) Find the value of the game.

Value of the game $= \dfrac{ad - bc}{a - b + d - c} = 0$

5. (6 marks) Solve the zero sum game with the game matrix

$$\begin{pmatrix} 4 & 5 & 2 & 0 \\ 3 & 1 & 3 & 6 \end{pmatrix}$$

Solution:



Reduce the game into

$$\begin{pmatrix} 5 & 2 \\ 1 & 3 \end{pmatrix}$$

By solving the reduced game

We have :

The maximin strategy of the row player is $\left(\frac{2}{5}, \frac{3}{5}\right)$

the minimax strategy of the column player is $\left(0, \frac{1}{5}, \frac{4}{5}, 0\right)$

Value of the game $= \frac{13}{5}$

6. (6 marks) There is a number of chips on the table. Two players play a game with the following rules. In each turn, a player may remove 1, 2 or 6 chips from the table. The player who makes the last move wins.

   (a) Determine whether n is a P-position or an N-position for $n = 7, 8, 9, 10$.

   (b) Find a winning move for the first player if initially there are n chips for $n = 89,\ 90$.

(a)

0	1	2	3	4	5	6	7	8	9	10
P	N	N	P	N	N	N	P	N	N	P

(b) n is a P-position if and only if the reminder of n divided by 7 is 0 or 3

So 84, 87 are P-position

and for n = 89 winning move is $89 \rightarrow 87$

for n = 90 winning move $90 \longrightarrow 84$

7. (6 marks) Let $\oplus$ denotes the nim-sum.

   (a) Find $6 \oplus 11 \oplus 14$.

   (b) Find x if $x \oplus 13 = 11$

   (c) Find all winning moves of the game of nim from the position $(6, 11, 14)$.

(a) $6 = 110_2$

$11 = 1011_2$

$13 = 1110_2$

$$\begin{array}{rr} & 110_2 \\ \oplus & 1011_2 \\ \oplus & 1110_2 \\ \hline & 11_2 \end{array}$$

So $6 \oplus 11 \oplus 13 = 11_2 = 3$

(b) if $x \oplus 13 = 11$

then $x = x \oplus 13 \oplus 13 = 11 \oplus 13$

$13 = 1101_2$

$11 \oplus 13 = 110_2 = 6$

So $x = 6$

(c) we have (x, y, z) is p-position if and only if $x \oplus y \oplus z = 0$

and $(5, 11, 14)$, $(6, 8, 14)$, $(6, 11, 13)$ are p-positions and all can be moved from $(6, 11, 14)$

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